## Enzyme rhythms in model inducilator.speedy - spontaneous oscillations

Model name: inducilator

- o Optimisation problem
- Protein turnover time 1 s = 0.0167 min
- No external perturbation considered
- Scored quantity: v1
- Scored quantity: v2
- Scored quantity: v3
- State-averaged fitness
- No posttranslational rhythms allowed
- Standard frequency considered f : 0.5/s (period 2 s)
- o Model properties:
- inactive\_enzymes: 0
- balanced\_reference\_state: 1
- consider\_external\_rhythm: 0
- adaptive\_rhythm: 0
- spontaneous\_rhythm: 1
- spontaneous\_rhythm\_at\_omega: 1
- has\_spontaneous\_rhythm\_and\_inactive\_enzymes: 0
- o Beneficial self-induced oscillation found
- Maximum principal synergy found (in tested range) at frequency f =0.501/s (period 2 s)
- Maximum fitness found (in tested range) at frequency f =0.316/s (period 3.16 s)
- o Self-induced oscillations?
- Maximally self-induced oscillations (in tested range) at f = 0.501, principal synergy 0.0164
- Beneficial self-induced oscillations found at frequency f = 0.5/s (principal synergy = 0.0164)
- Predicted fitness change (self-induced, 2nd order, amplitude below 1/2 of mean) at frequency f =0.5: 0.0369

- Predicted maximal fitness change (self-induced, numeric opt, full amplitude constraints) at frequency f =0.316: 0.0272

o Numerical calculation (self-induced rhythm, amplitude below 1/2 of mean, f=0.5)

- Fitness change (fitness-averaged) : -0.0106
- Fitness change (state-averaged): 0.0132



Figure 1: Network and reference flux



Figure 2: Reference state (top) and mean state during oscillation (bottom).



Figure 3: Control analysis: fitness curvatures. Left: Frequency-dependent fitness curvature eigenvalues. Right: relative sizes and phases of the individual enzyme levels (components of the leading fitness curvature eigenvector).



Protein level and enzyme activity (mean and amplitude)



Figure 5: Numerical calculations: spontaneous oscillations. Perturbation frequency see first page.



Figure 6: Spontaneous oscillations (local expansion; arrows: absolute changes). Perturbation frequency see first page.



Figure 7: Spontaneous oscillations (or tendencies towards them) for various circular frequencies  $\omega$ . If the maximal fitness curvatures  $\lambda$  is positive, the rhythm is beneficial (local expansion; arrows: absolute changes).