

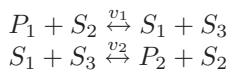
# Vorlesung “Modellierung von Zellprozessen”

## Aufgabenblatt 1: Modellierung

WS 2009/2010, Dozent: Wolfram Liebermeister

### 1. Small network structure

Draw the the reaction system



as a network, write down its stoichiometric matrix, and determine potential fluxes in stationary state. Are there linear conservation relations? ( $v_i$ : reaction rate,  $S_j$ : internal (variable) compounds,  $P_k$ : external (fixed) compounds)

### 2. From the ODEs back to the reaction

Consider the following system of differential equations. Try to find out a reaction scheme that corresponds to these reactions. Are there cases of activation or inhibition?

$$\begin{aligned} \frac{dM_1^-}{dt} &= -k_1 \cdot M_1^- \cdot \frac{1}{(1 + M_3^+)} + k_2 \cdot M_1^+ \\ \frac{dM_1^+}{dt} &= +k_1 \cdot M_1^- \cdot \frac{1}{(1 + M_3^+)} - k_2 \cdot M_1^+ \\ \frac{dM_2^-}{dt} &= -k_3 \cdot M_2^- \cdot M_1^+ + k_4 \cdot M_2^+ \\ \frac{dM_2^+}{dt} &= +k_3 \cdot M_2^- \cdot M_1^+ - k_4 \cdot M_2^+ \\ \frac{dM_3^-}{dt} &= -k_5 \cdot M_3^- \cdot M_2^+ + k_6 \cdot M_3^+ \\ \frac{dM_3^+}{dt} &= +k_5 \cdot M_3^- \cdot M_2^+ - k_6 \cdot M_3^+ \end{aligned}$$

**3. The repressilator** The repressilator (M. Elowitz and S. Leibler, Nature 2000) is a genetic circuit consisting of three proteins, each inhibiting the production of the following protein in a circle. Consider the following kinetic equations for synthesis and degradation of the proteins:

$$v_i^{\text{syn}} = \frac{\beta}{1 + x_{l(i)}/k}$$

$$v_i^{\text{degr}} = \alpha x_i$$

with  $i = 1, 2, 3$  and  $l(1) = 3, l(2) = 1, l(3) = 2$ .

The reaction rate vector reads  $v = (v_1^{\text{syn}}, v_2^{\text{syn}}, v_3^{\text{syn}}, v_1^{\text{degr}}, v_2^{\text{degr}}, v_3^{\text{degr}})^T$ .

(a) Write down the stoichiometric matrix and the differential equations for the protein levels  $x_i$ .

(b) Consider a stationary state with  $x_1 = x_2 = x_3 = \bar{x}$ . Calculate the elasticity matrix  $\epsilon^S$ . (Hint: it is not necessary to compute the value of  $\bar{x}$ .)

(c) Compute the Jacobian matrix by the formula  $M = N\epsilon^S$ .

(d) How could you decide based on  $M$  whether the stationary state is stable?