Vorlesung “Modellierung von Zellprozessen”
Aufgabenblatt 3: Genregulation

WS 2009/2010, Dozent: Wolfram Liebermeister

1. **Kinetic model of transcription and translation.** Show that the kinetic model for concentrations c_x, c_y

$$\begin{aligned} \dot{c}_x &= 1 - c_x \\ \dot{c}_y &= c_x - c_y \end{aligned} \tag{1}$$

with initial conditions $c_x(0) = c_x^*, c_y(0) = c_y^*$ is solved by

$$\begin{aligned} c_x &= 1 + (c_x^* - 1)\,\mathrm{e}^{-t} \\ c_y &= 1 + \big((c_y^* - 1) + (c_x^* - 1)\,t\big)\ \mathrm{e}^{-t}. \end{aligned} \tag{2}$$

Sketch the solutions for different the following pairs of initial values (c_x^*, c_y^*): (0,0), (1,0), (0,1), (1,1). Hint: compute the steady state and introduce new variables $\Delta c_x(t), \Delta c_y(t)$ representing the respective deviations from the steady state. Compute the deviation $\Delta c_x(t)$ first and then use the ansatz $\Delta c_y(t) = (\alpha + \beta t)\mathrm{e}^{-t}$.

2. **Cooperativity** Consider a transcription factor that exists both in the form of unbound monomers (called X) and bound to DNA as n-mers (called C):

$$n\ X + D \leftrightarrow C \tag{3}$$

D_{tot} denotes the DNA binding sites, D the free DNA binding sites. Show that in chemical equilibrium,

$$\begin{aligned} D &= \frac{D_{\mathrm{tot}}}{1 + (X/K_{\mathrm{D}})^n} \\ C &= \frac{D_{\mathrm{tot}}\ X^n}{K_{\mathrm{D}}^n + X^n} \end{aligned} \tag{4}$$

where K_{D} is the equilibrium constant and all other quantities are the respective concentrations.

3. **Two binding sites.** Consider a promoter with two identical binding sites for an activator. The gene is transcribed with a rate v_1 if both sites are occupied, and with rate 0 otherwise. Draw the gene input function (average transcription rate y as a function of the free activator concentration x); assume that there is no cooperativity (energetic coupling) between activators bound at the two binding sites.

4. **Logic gates** How can an AND or OR logic be realized by transcription factor binding?