PERIODIC CORPUSCLE STRUCTURES AND THE SPACES IN BETWEEN

Eva WOHLLEBEN[1], Felix HEDIGER[2], Wolfram LIEBERMEISTER[3]

[1]Muthesius-Kunsthochschule Kiel, Germany
[2]Artist – www.kuenstlermensch.de
[3]Charité - Universitätsmedizin Berlin, Germany

ABSTRACT: Corpuscles are flexible geometric elements formed by regular triangles. Corpuscle elements can be joined, thus serving as building blocks for polyhedral chains and complex three-dimensional structures. Here we describe a periodic three-dimensional corpuscle grid whose negative space consists of periodically arranged octahedra. As a recurrent local structure, the grid contains the octahedron-centric corpuscle ball, an arrangement of 18 corpuscle elements surrounding an empty, octahedron-shaped space. Similarly, we construct corpuscle balls with empty cubes or icosahedra at their centres and the according rotation symmetries. Both corpuscle balls can be extended to form periodic grids, but only if slight deformations of the edge lengths are allowed. The corpuscle grids and balls contain rigid, ring-like substructures, which stabilise them against deformation.

Keywords: Corpuscle, flexible polyhedron, tiling

1. INTRODUCTION Corpuscles are flexible geometric elements that can serve as building blocks for polyhedral clusters, chains, and periodic three-dimensional grids [1]. Each corpuscle consists of several triangles connected by flexible edges. Between these triangles, there may be open slits, called mouths, which permit to link several corpuscle elements. Some of the resulting larger structures allow for flexible collective movements, especially if some of the mouths are left open. Also some closed structures, like Goldberg's “Siamese double pyramid” icosahedron [3], can flex with little deformation of the triangle edge lengths. Closed rings, formed by chains of corpuscle elements, are rigid unless the ring size (number of elements forming the ring) is a multiple of three [2]. All this becomes apparent in paper models.

In [1], we presented a grid-like corpuscle structure that extends periodically throughout the entire three-dimensional space. Here we describe this structure in terms of two architectures: one architecture, the “positive” corpuscle grid itself, is formed by the interconnected corpuscles. The second architecture is formed by the empty “negative” space in between. Its shape is obtained by stacking regular solids and antiprisms. Antiprisms, like normal prisms, have regular polygons at their bottom and top faces, but tilted against each other and connected by regular triangles. The construction via the negative space does not account for possible flexibility, but makes it easier to see if the structures can actually be periodic, and if deformations are necessary. Taken together, both grids fill the space completely. They are separated by a membrane consisting of triangles: on its one side, it forms the corpuscle faces, and on the other, the band of triangles surrounding the anti-prisms.

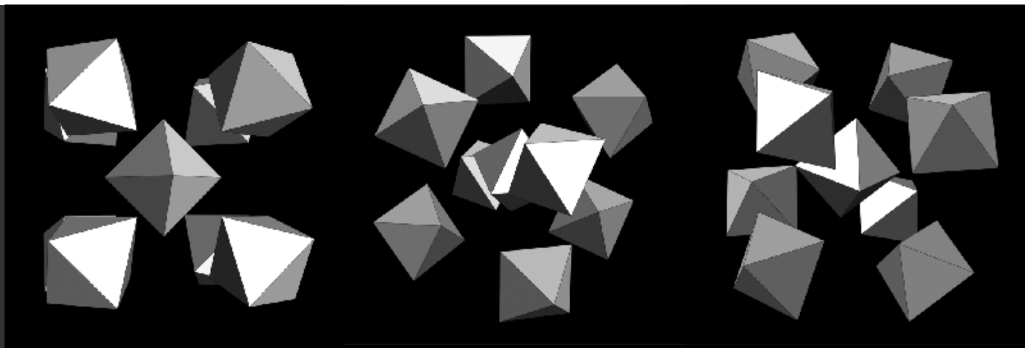
After developing this structure from a central empty octahedron, we present structures with other rotation symmetries based on an empty cube or icosahedron as their central spaces. Also these structures can be periodically extended, but slight deformations of the edge lengths are necessary. To build the positive corpuscle grid, we arrange corpuscle elements around the empty centre and add more and more elements mouth by mouth. Instead of continuing this process, we can also close open mouths by triangles, which leads to closed corpuscle balls. Again, this may require slight deformations of the edge lengths [2]. Exact structures, which consist of regular triangles, will be shown by computer drawings, while paper models show structures that require some elastic deformation. Also the membrane between the positive and the negative space, which can contain regular or irregular triangles, is shown by paper models.

2. OCTAHEDRON-CENTRIC CORPUSCLE GRID

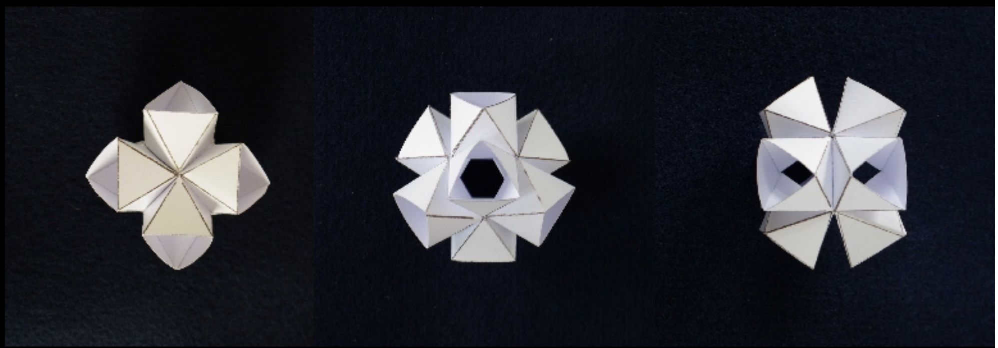
The construction of the octahedron-centric corpuscle grid is shown in Figure 1a-e. We first consider the negative grid, in which regular octahedra are stacked along their 3-fold symmetry axes. To understand its local structure, we first consider a central octahedron, to whose faces other octahedra are attached face to face, thus giving them a different orientation (Figure 1 a-b). To each of these octahedra, we could attach another octahedron, which then would have the same orientation as the initial, central octahedron. This stacking can be continued, leading to a periodic grid with cubic symmetry: we can see this by nothing that the first shell of octahedra fits exactly into a cubic elementary cell. Eventually, we obtain a periodic alternation of two types of octahedra: those which are surrounded by octahedra at all of their faces (type A); and those that have only two direct neighbours (type B). For reasons that will become clear below, the type B octahedra are also called "triangular antiprisms".
The octahedron-centric grid can be seen as the negative space of the actual corpuscle grid, which is shown now. Each corpuscle in the grid consists of four double-triangles ("segments"), surrounding a central axis and with mouths in between them. These mouths leave space to fit in other corpuscle elements with the same shape, but different orientations. By adding more and more corpuscle elements, shell by shell surrounding the central solid, the structure can be extended to infinity. Positive and negative grid are separated by a membrane consisting of triangles. This membrane, and therefore all corpuscle faces, are formed by faces of the type B octahedra.

Figure 1a-e: Negative space of the octahedron-centric corpuscle grid. (a) An octahedron (type A) is surrounded by eight other octahedra in a different orientation (type B). The arrangement is shown from three different angles, highlighting the 4-fold (left), 3-fold (centre), and 2-fold (right) symmetry axes. (b) The surrounding octahedra are now attached to the faces of the central octahedron. Their outer faces, and those pointing towards the central octahedron, will not be part of the membrane and are therefore not shown in the models from this row on. (c) The arrangement of octahedra fits exactly into a cubic cell; to show this more clearly, pyramids are attached to the openings. If the cubic cell is periodically extended, we obtain a grid with cubic symmetry. (d) In this model, eight pyramids surround each cube corner, forming an octahedron in the same orientation as the central one.

1a



1b



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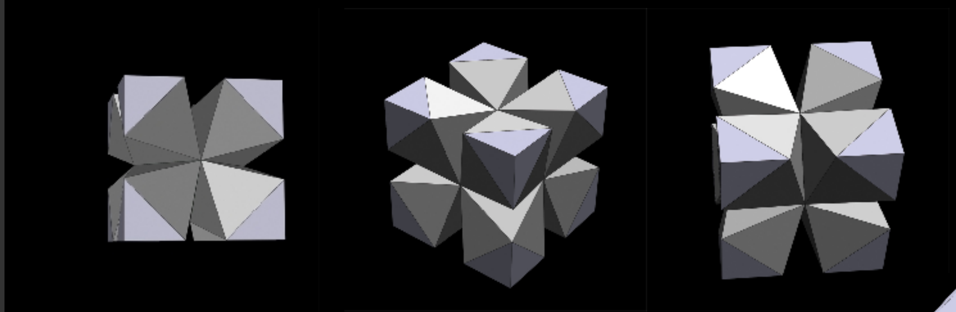
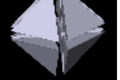
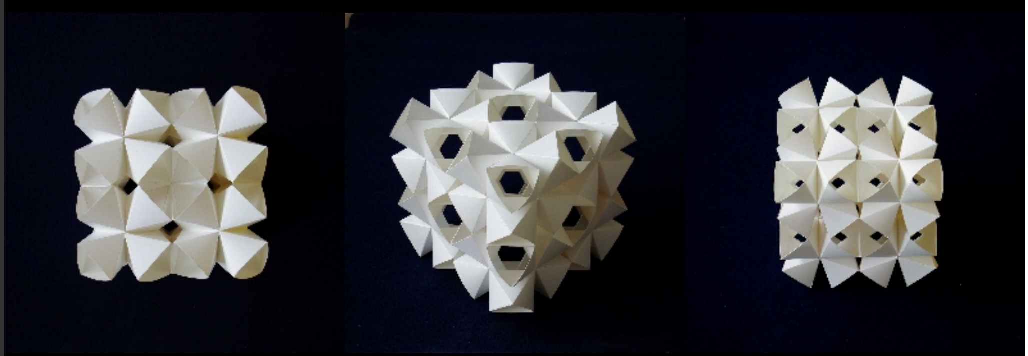


1d



1e



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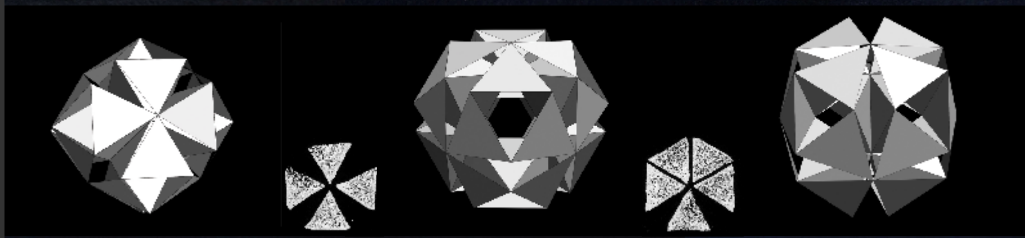
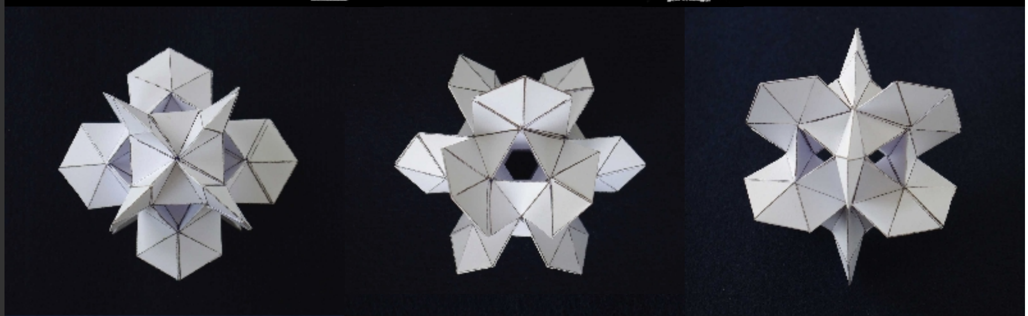
1f

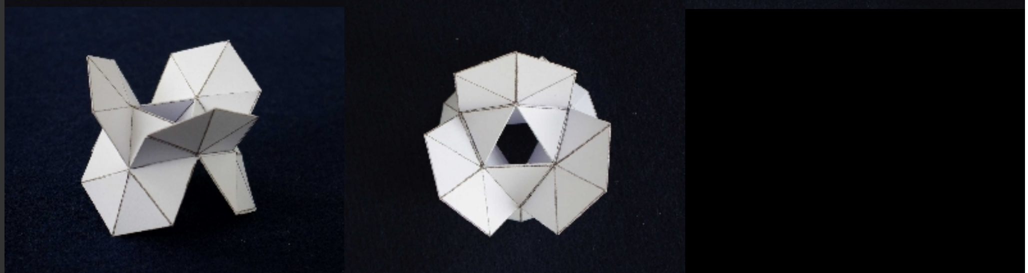
1g



1h



1i



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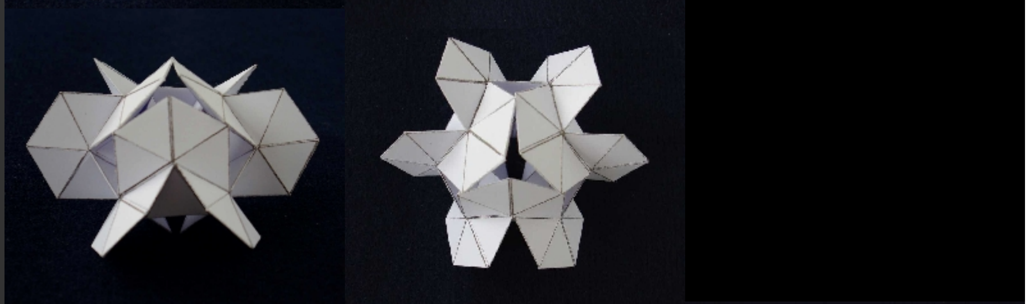


1k

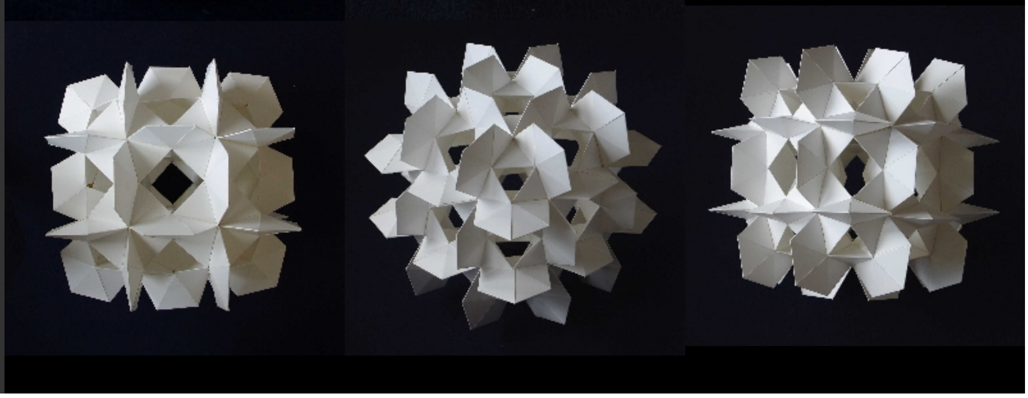
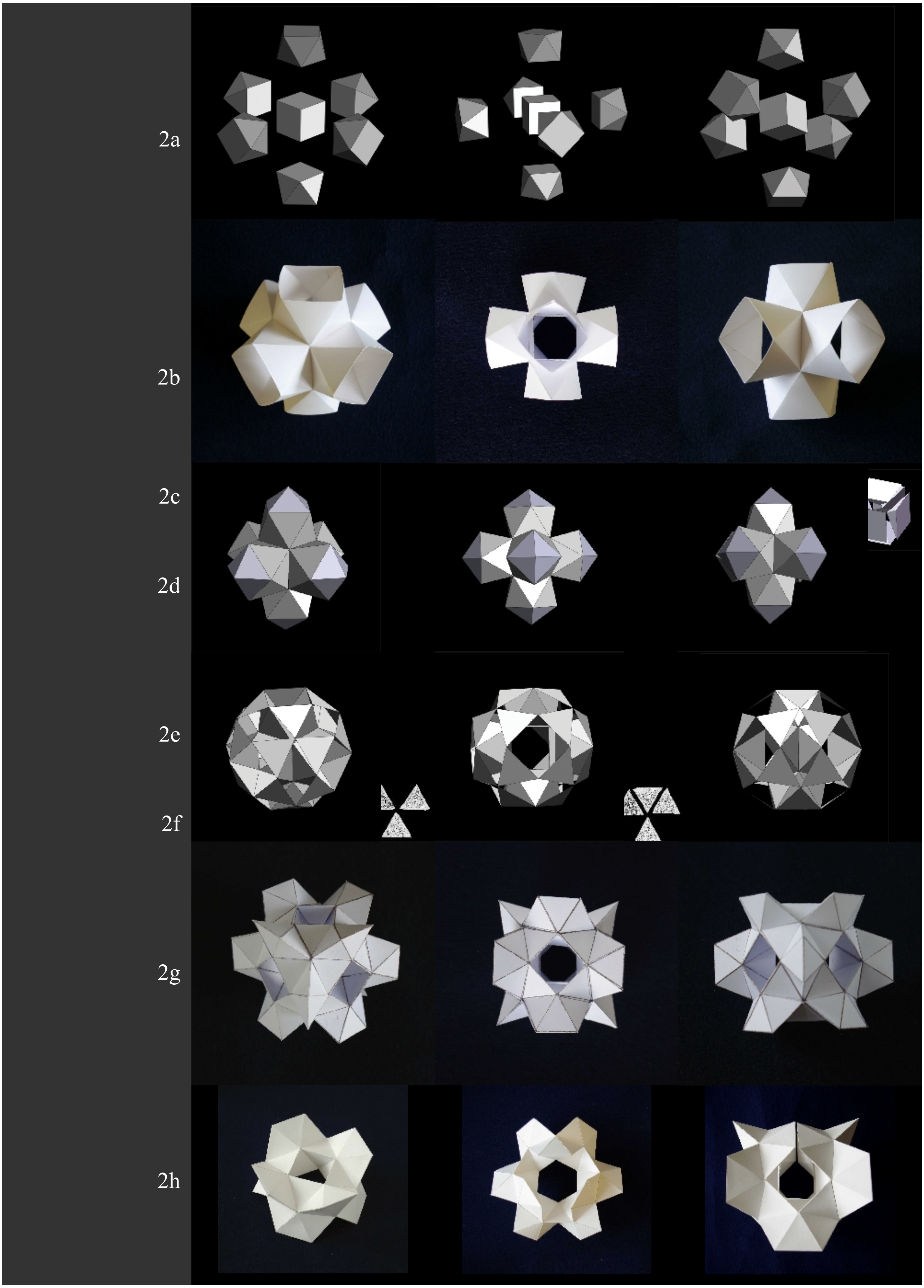


Figure 1f-k: Positive space of the octahedron-centric corpuscle grid. Each type B octahedron forms the rear side of three corpuscles from the first shell and of three corpuscles from the second shell. Corpuscles of the first shell contact the vertices of the central octahedron; corpuscles from the second shell contact the edges of the central octahedron. Yet all corpuscles have open mouths. (f) The corpuscles from the first shell are completed. (h) The corpuscles of the second shell are completed, and their two remaining mouths are closed by additional triangles. The positive space has become an enclosed volume, and the membrane is now the surface of a polyhedron. We obtain the octahedron-centric corpuscle ball (h): the empty octahedron is surrounded by six regular corpuscles (each with 4 segments, all separated by mouths), which are connected by twelve more, slightly deformed corpuscles (1 segment plus 4 adjacent segments). Eight tunnels lead from the empty centre to the outside space. Since the octahedron in the centre is rigid, the octahedron ball and the entire corpuscle grid are rigid as well. (i) We choose a band of corpuscles that surrounds one of the tunnels of the octahedron-centric corpuscle ball, treat the first-shell corpuscles like the second-shell corpuscles before, and extract a flexible substructure, comprising six elements with 5 segments and 2 mouths each. (j) A longer band of elements is extracted from the ball, leading to a flexible ring structure, comprising 12 elements of the same type. (k) relates to (e): the structure shown extracts the fourth and fifth shell of corpuscles from an octahedron-centric grid. By closing the remaining mouths – towards the centre without an additional triangle, towards outside with one additional triangle – we obtain a branched polyhedron comprising 48 elements of the 4th shell – corpuscles with four segments and four mouths each - and 24 elements of the 5th shell – corpuscles with 5 segments and 2 mouths.

3. CUBE-CENTRIC CORPUSCLE GRID

By choosing a cube, instead of an octahedron, as our central empty space, we obtain another type of corpuscle grid. For the negative grid, we connect cubes (type A solids) by square antiprisms (type B solids). The resulting module (Fig. 2d) fits almost, but not precisely, into an octahedron. If the fit was exact, these octahedra could be stacked periodically according to the scheme in Fig. 1a-d. In reality, this stacking is only possible with a slight deformation of the triangle edge lengths. This time, the positive space is formed by corpuscle elements with 3 segments each. If we extract and close a basic corpuscle arrangement, we obtain the cube-centric corpuscle ball (Fig. 2g). It surrounds a central empty cube with empty antiprisms on all faces, which serve as the tunnels to the outside space. Moreover, the corpuscle structure contains corpuscle rings composed of 8, 12, or 16 elements (Fig. 2h). Among these rings, only the 12-ring is flexible [2].

Figure 2: Cube-centric corpuscle grid. The construction is analogous to the one shown in Figure 1, but based on an empty cube at the centre. (a-b) A cube is surrounded by a first shell of antiprisms. The antiprisms have squares at their top and bottom, which are tilted against each other and connected by a ring of regular triangles. The structure in (d) fits almost, but not exactly, into a Platonic octahedron. Otherwise, it could be periodically stacked according to periodic octahedra stacking shown in Figure 1. The closed structure in (g) is the cubic corpuscle ball described in [1]. (h) Three different ring structures contained in the cubic corpuscle ball, with 8, 12 and 16 units.


2a
2b
2c
2d
2e
2f
2g
2h

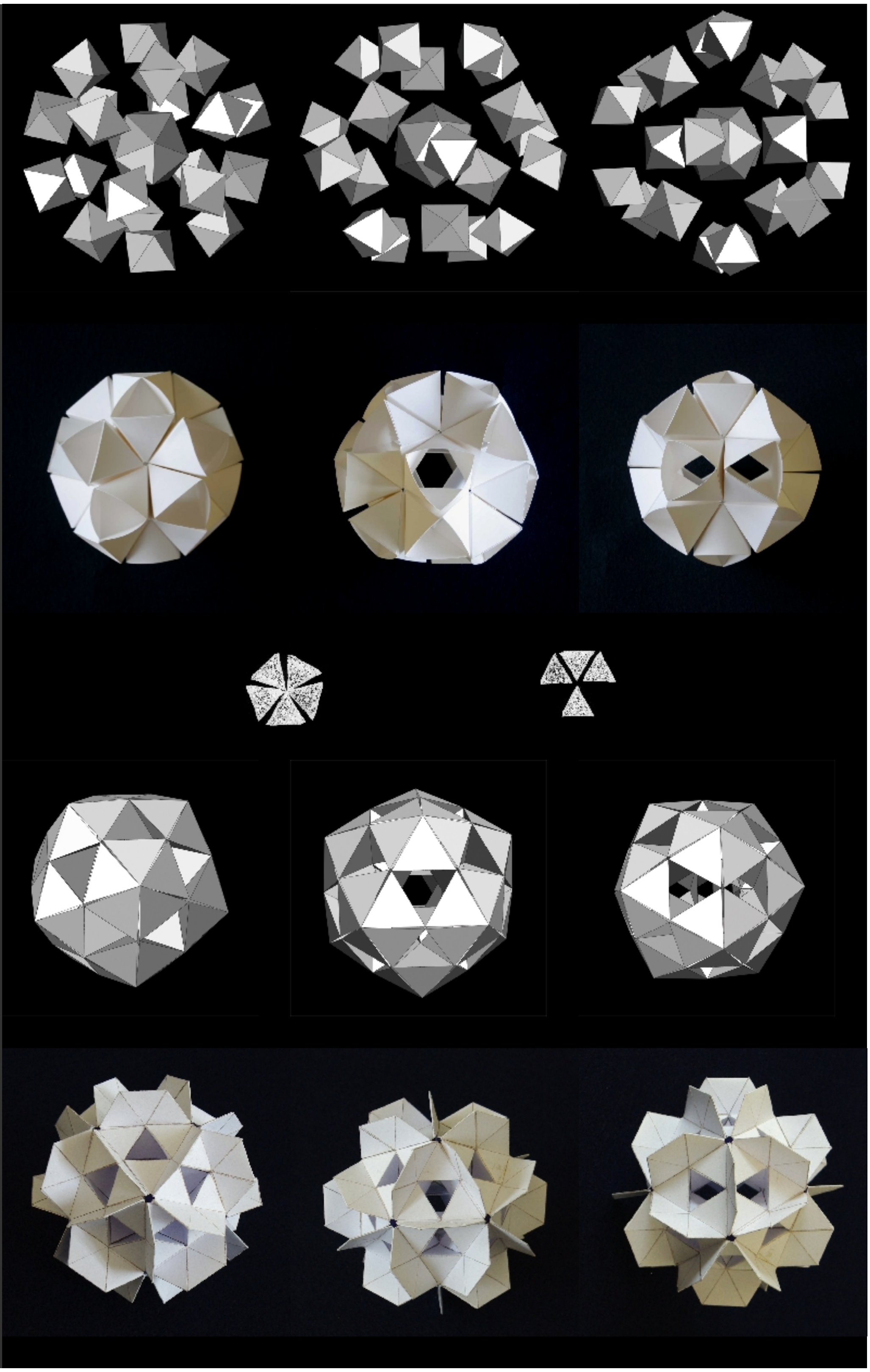
4. ICOSAHEDRON-CENTRIC CORPUSCLE BALL

To create a corpuscle ball with icosahedral symmetry, we surround an icosahedron by a shell of octahedra, that is, triangle antiprisms (Fig. 3a-b). If this empty space is surrounded by corpuscles, an icosahedron corpuscle ball emerges. Again, the edges towards the outside have to be slightly stretched. In (Fig. 3d), the twelve corpuscles of the first shell are completed. They consist of 5 segments and 5 mouths each. After adding a second shell of corpuscle elements - 4 segments and 2 mouths each - the icosahedral corpuscle ball appears (Fig. 3e). It contains thirty 3-segment bridges, which point outwards and are almost flat. To further expand this network, we think of the negative space again, add tetrahedra to the outer faces of the antiprisms, and surround these tetrahedra again by antiprisms; all these antiprisms happen to be octahedra. The emerging, more complex icosahedric corpuscle ball contains four shells of corpusles and consists of alternating network corpuscle elements with 3 or 5 mouths (Fig. 3g). Since the emerging tunnels are strongly curved, the central empty icosahedron is not visible anymore from the outside.

As we expand the paper models shell by shell, tension builds up and any further expansion becomes tough. We can reduce the tension in the third and fourth shell by removing the elements at the core, i.e. its first and second corpuscle shells. Similar to the structure shown in Fig. 1k, remaining mouths pointing towards the inside can be closed directly without adding new triangles. The resulting structure shows relatively little deformation: it comprises 20 elements with 3 segments and 3 mouths each, almost flat, and thirty rather bold elements, each with 5 segments and 2 mouths.

Like in the case of the cube-centric grid module, this module could be repeated periodically within a higher-level structure. Again, this higher-level structure can consist of a Platonic solid and an antiprism – in this case, the dodecahedron and the pentagonal antiprism – stacked in alternation. Among the twelve pentagon faces of a dodecahedron, only six are covered by antiprisms, while the other six remain uncovered (Fig. 3j). For the icosahedron-centric ball, periodic and quasiperiodic extensions to infinity could be imagined, but have not yet been shown to exist.

Figure 3: Icosahedron-centric corpuscle ball. (a-c) A Platonic icosahedron is surrounded by octahedra (the antiprisms corresponding to its triangle faces). The construction resembles the one shown in Figures 1 and 2. (d) The first shell of corpuscles is completed. (e) By completing and closing the second shell of corpuscles, we obtain the icosahedron-centric corpuscle ball. (f) A layer of tetrahedra, surrounded by antiprisms (g) is attached to the central icosahedron. Together, these polyhedra build the negative space of complex icosahedron balls, here containing the first four shells. In (h), the first and second shells have been removed. (j) The Platonic dodecahedron, in alternation with pentagonal antiprisms, can serve as a higher-level structure in which the coreless ball shown in (h) as combined with (i) a corpuscle structure surrounding a pentagonal antiprism (10 network corpuscles with 4 mouths and 5 segments, and 20 chain corpuscles with 2 mouths and 4 segments each).



3a

3b

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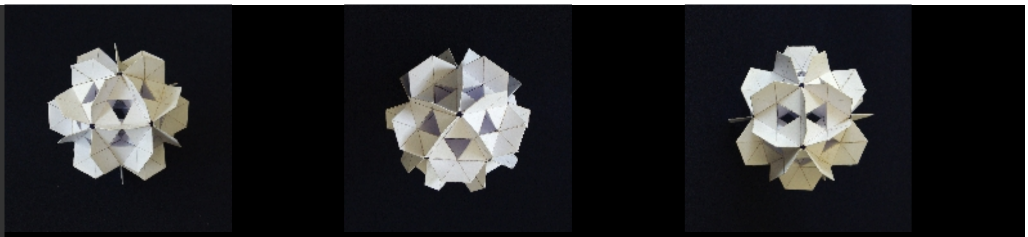

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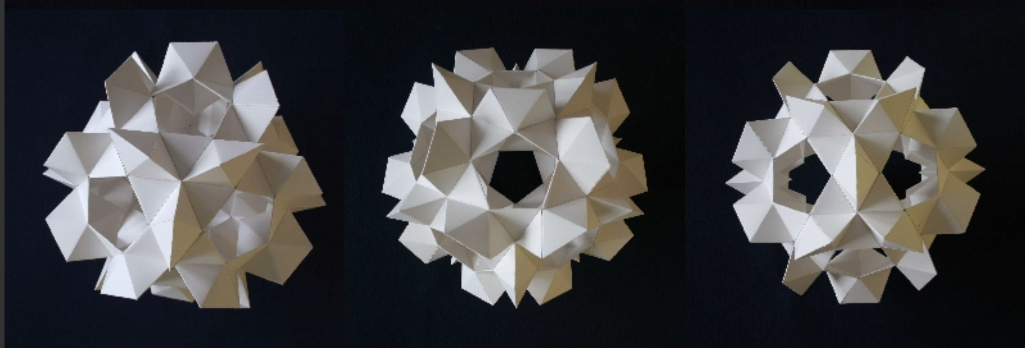

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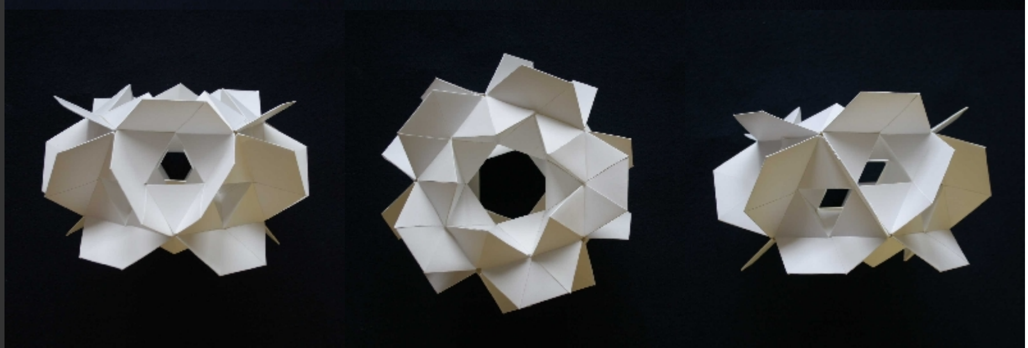

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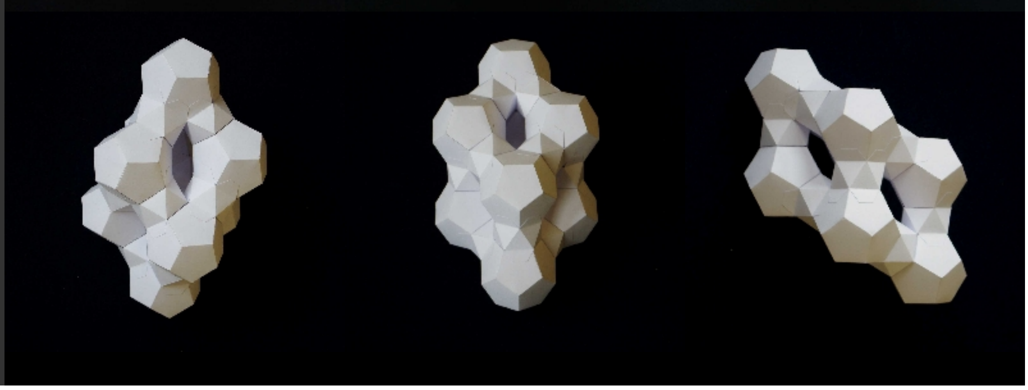

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ABOUT THE AUTHORS

Eva Wohlleben uses artistic strategies to explore geometry in motion. She visualizes pure logic in space, time and material. Her interest is focused on the verge between dual structures and possible transformations of that. For monades with a definite flexibility, she coined the term "corpuscle geometry".

website: www.korpuskel.de.

Wolfram Liebermeister is a physicist and holds a doctorate degree in theoretical biophysics. For his diploma thesis, he studied the atomic structure of icosahedral quasicrystals. His research in systems biology is focused on mathematical modelling of living cells, including uncertainty and variability analysis, control theory, and studies of information processing in cells.

website: http://jaguar.biologie.hu-berlin.de/~wolfram

Felix Hediger is artist and design engineer. His aim is to apply princips of living nature to technical design. Therefor he explores geometric forms in nature, especially screws, spirals and vortexes, and uses their geometric foundation, such as the golden mean and inversion, to create machines for water treatment, energy collection and energy transformation.

website: www.kuenstlermensch.de